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Artificial Intelligence II

Exercise 2

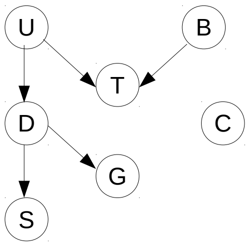
Q1. Bayes' Nets Modeling

Consider the following domain:

- Urbanization (U)
- Deforestation (D)
- Soil erosion (S)
- Global warming (G)
- Traffic (T)
- Cavity (C)
- Baseball game (B)

(a) Construct the Bayes net from the above domain. **U, D, S, G, T, C** and **B** represent random variables of this domain. There can be more than one ways to model the Bayes net from the domain above, but you should have an explanation for your answer.

Solution:



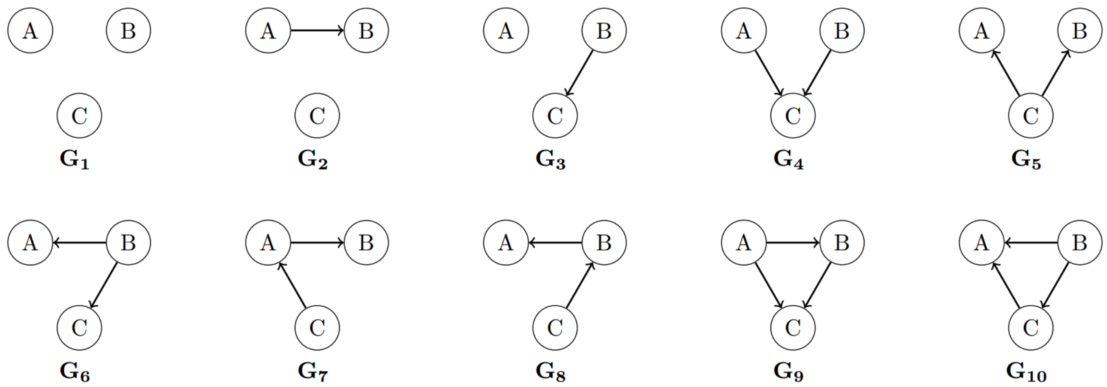
Urbanization(U) causes Deforestation(D) which causes Soil erosion (S). Deforestation(D) is one of the causes of Global warming(G). Both Urbanization(U) and Baseball match(B) can cause Traffic(T). Cavity(C) does not seem related to any of the above events.

(b) Define the CPT (Conditional Probability Table) for each node (random variable) in your Bayes network topology *(Hint: CPT for U can be defined as **P(U)**)*

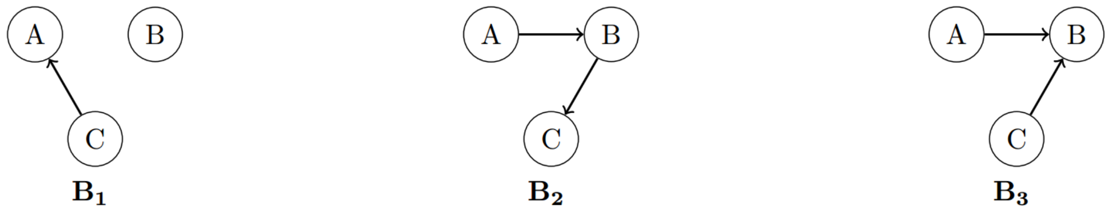
> **Solution:**
>
> In the Bayes net above, CPT for U, D, S, G, T, B and C are P(U), P(D | U), P(S | D), P(G | D), P(T | U,B) and P(C) respectively.

Q2. Bayes' Nets Representation and Conditional Independence

Assume we are given the following ten Bayes' nets, labeled G_1 to G_{10}:



Assume we are also given the following three Bayes' nets, labeled B_1 to B_3:



> **Solution:**
>
> Before we go into the questions, let's enumerate all of the (conditional) independence assumptions encoded in all the Bayes' nets above. They are:
>
> - G_1 : $AB; AB|C; AC; AC|B; BC; BC|A$
> - G_2 : $AC; AC|B; BC; BC|A$
> - G_3 : $AB; AB|C; AC; AC|B$
> - G_4 : AB

- G_5 : $A \perp B | C$
- G_6 : $A \perp C | B$
- G_7 : $B \perp C | A$
- G_8 : $A \perp C | B$
- G_9 : $\emptyset$
- G_{10} : $\emptyset$
- B_1 : $A \perp B; A \perp B | C; B \perp C; B \perp C | A$
- B_2 : $A \perp C | B$
- B_3 : $A \perp C$

(a) Assume we know that a joint distribution d_1 (over $A; B; C$) can be represented by Bayes' net B_1. Mark all of the following Bayes' nets that are guaranteed to be able to represent d_1.

☐ G_1	☐ G_2	☐ G_3	■ G_4	■ G_5
☐ G_6	■ G_7	☐ G_8	■ G_9	■ G_{10}

☐ None of the above.

Solution:

Since B_1 can represent d_1, we know that d_1 must satisfy the assumptions that B_1 follows, which are: $A \perp B; A \perp B | C; B \perp C; B \perp C | A$. We cannot assume that d_1 satisfies the other two assumptions, which are $A \perp C$ and $A \perp C | B$, and so a Bayes' net that makes at least one of these two extra assumptions will not be guaranteed to be able to represent d_1. This eliminates the choices $G_1; G_2; G_3; G_6; G_8$. The other choices $G_4; G_5; G_7; G_9; G_{10}$ are guaranteed to be able to represent d_1 because they do not make any additional independence assumptions that B_1 makes.

(b) Assume we know that a joint distribution d_2 (over $A; B; C$) can be represented by Bayes' net B_2. Mark all of the following Bayes' nets that are guaranteed to be able to represent $d2$.

☐ G_1	☐ G_2	☐ G_3	☐ G_4	☐ G_5
■ G_6	☐ G_7	■ G_8	■ G_9	■ G_{10}

☐ None of the above.

Solution:

Since B_2 can represent d_2, we know that d_2 must satisfy the assumptions that B_2 follows, which is just: $AC|B$. We cannot assume that d_2 satisfies any other assumptions, and so a Bayes' net that makes at least one other extra assumptions will not be guaranteed to be able to represent d_2. This eliminates the choices $G_1; G_2; G_3; G_4; G_5; G_7$. The other choices $G_6; G_8; G_9; G_{10}$ are guaranteed to be able to represent d_2 because they do not make any additional independence assumptions that B_2 makes.

(c) Assume we know that a joint distribution d_3 (over $A; B; C$) **cannot** be represented by Bayes' net B_3. Mark all of the following Bayes' nets that are guaranteed to be able to represent d_3.

□ G_1	□ G_2	□ G_3	□ G_4	□ G_5
□ G_6	□ G_7	□ G_8	■ G_9	■ G_{10}

□ None of the above.

Solution:

Since B_3 cannot represent d_3, we know that d_3 is unable to satisfy at least one of the assumptions that B_3 follows. Since B_3 only makes one independence assumption, which is AC, we know that d_3 does not satisfy AC. However, we can't claim anything about whether or not d_3 makes any of the other independence assumptions. d_3 might not make any (conditional) independence assumptions at all, and so only the Bayes' nets that don't make any assumptions will be guaranteed to be able to represent d_3. Hence, the answers are the fully connected Bayes' nets, which are $G_9; G_{10}$.

(d) Assume we know that a joint distribution d_4 (over $A; B; C$) can be represented by Bayes' nets B_1, B_2, and B_3. Mark all of the following Bayes' nets that are guaranteed to be able to represent d_4.

■ G_1	■ G_2	■ G_3	■ G_4	■ G_5
■ G_6	■ G_7	■ G_8	■ G_9	■ G_{10}

□ None of the above.

Solution:

Since $B_1; B_2; B_3$ can represent d_4, we know that d_4 must satisfy the assumptions that $B_1; B_2; B_3$ make. The union of assumptions made by these Bayes' nets are: $AB; AB|C; BC; BC|A; AC; AC|B$. Note that this set of assumptions encompasses all the possible assumptions that you can make with 3 random variables, so any Bayes' net over $A; B; C$ will be able to represent d_4.