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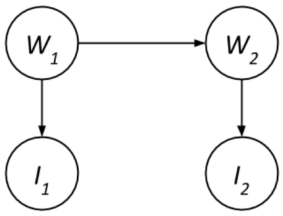
Artificial Intelligence II

Exercise 4

Q1. Sampling and Dynamic Bayes Nets

We would like to analyze people's ice cream eating habits on sunny and rainy days Suppose we consider the weather, along with a person's ice-cream eating, over the span of two days. We'll have four random variables: W_1 and W_2 stand for the weather on days 1 and 2, which can either be rainy R or sunny S, and the variables I_1 and I_2 represent whether or not the person ate ice cream on days 1 and 2, and take values T (for truly eating ice cream) or F. We can model this as the following Bayes Net with these probabilities.



W_1	$P(W_1)$
S	0.6
R	0.4

W_1	W_2	$P(W_2\|W_1)$
S	S	0.7
S	R	0.3
R	S	0.5
R	R	0.5

W	I	$P(I\|W)$
S	T	0.9
S	F	0.1
R	T	0.2
R	F	0.8

Suppose we produce the following samples of (W_1, I_1, W_2, I_2) from the ice-cream model:

~~R,F,R,F~~ ~~R,F,R,F~~ ~~S,F,S,T~~ ~~S,T,S,T~~ S,T,R,F
~~R,F,R,T~~ ~~S,T,S,T~~ ~~S,T,S,T~~ S,T,R,F ~~R,F,S,T~~

1. What is $\hat{P}(W_2 = R)$, the probability that sampling assigns to the event $W_2 = R$?

Solution:

Number of samples in which $W_2 = R$: 5. Total number of samples: 10. Answer $5/10 = 0.5$.

2. Cross off samples above which are rejected by rejection sampling if we're computing $P(W_2|I_1 = T, I_2 = F)$.

Rejection sampling seems to be wasting a lot of effort, so we decide to switch to likelihood weighting. Assume we generate the following six samples given the evidence $I_1 = T$ and $I_2 = F$:

$$(W_1, I_1, W_2, I_2) = \{(S,T,R,F), (R,T,R,F), (S,T,R,F), (S,T,S,F), (S,T,S,F), (R,T,S,F)\}$$

3. What is the weight of the first sample (S,T,R,F) above?

Solution:

The weight given to a sample in likelihood weighting is

$$\prod_{\text{Evidence variables } e} Pr(e|Parents(e)).$$

In this case, the evidence is $I_1 = T$, $I_2 = F$. The weight of the first sample is therefore

$$w = Pr(I_1 = T|W_1 = S) \cdot Pr(I_2 = F|W_2 = R) = 0.9 \cdot 0.8 = 0.72$$

4. Use likelihood weighting to estimate $P(W_2|I_1 = T, I_2 = F)$.

Solution:

The sample weights are given by

(W_1, I_1, W_2, I_2)	w
S,T,R,F	0.72
R,T,R,F	0.16
S,T,R,F	0.72

(W_1, I_1, W_2, I_2)	w
S,T,S,F	0.09
S,T,S,F	0.09
R,T,S,F	0.02

To compute the probabilities, we thus normalize the weights and find

$$\hat{P}(W_2 = R|I_1 = T, I_2 = F) = \frac{0.72+0.16+0.72}{0.72+0.16+0.72+0.09+0.09+0.02} = 0.889$$

$$\hat{P}(W_2 = S|I_1 = T, I_2 = F) = 1 - 0.889 = 0.111.$$