Prof. Wolfgang Nejdl
M.Sc. Ambreen Zaina, M.Sc. Jeff Reimer
Department of Knowledge-based Systems (KBS)

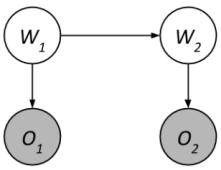
December 16, 2019

Artificial Intelligence II

Exercise 6

Q1. HMMs

Consider the following Hidden Markov Model.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}\|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that we observe $O_1 = A$ and $O_2 = B$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = A, O_2 = B)$ one step at a time.

1. Compute $P(W_1, O_1 = A)$.

Solution:

2. Using the previous calculation, compute $P(W_2, O_1 = A)$.

Solution:

3. Using the previous calculation, compute $P(W_2, O_1 = A, O_2 = B)$.

Solution:

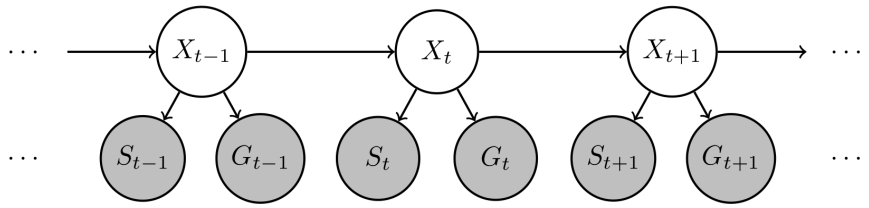
4. Finally, compute $P(W_2|O_1 = A, O_2 = B)$.

Solution:

Q2. HMM: Where is the Car?

Transportation researchers are trying to improve traffic in the city but, in order to do that, they first need to estimate the location of each of the cars in the city. They need our help to model this problem as an inference problem of an HMM. For this question, assume that only *one* car is being modeled.

(a) The structure of this modified HMM is given below, which includes X, the location of the car; S, the noisy location of the car from the signal strength at a nearby cell phone tower; and G, the noisy location of the car from GPS.



We want to perform filtering with this HMM. That is, we want to compute the belief $P(x_t|s_{1:t}, g_{1:t})$, the probability of a state x_t given all past and current observations.

The **dynamics update** expression has the following form:

$$P(x_t|s_{1:t-1}, g_{1:t-1}) = \underline{\textbf{(i)}} \quad \underline{\textbf{(ii)}} \quad \underline{\textbf{(iii)}} \; P(x_{t-1}|s_{1:t-1}, g_{1:t-1})$$

Complete the expression by choosing the option that fills in each blank.

(i)	□ $P(s_{1:t}, g_{1:t})$	□ $P(s_{1:t-1}, g_{1:t-1})$	□ $P(s_{1:t-1})P(g_{1:t-1})$
	□ $P(s_{1:t})P(g_{1:t})$	□ 1	
(ii)	□ $\sum_{x_t}$	□ $\sum_{x_{t-1}}$	□ $max_{x_{t-1}}$
	□ max_{x_t}	□ 1	
(iii)	□ $P(x_{t-1}\|x_{t-2})$	□ $P(x_{t-2}, x_{t-1})$	□ $P(x_{t-1}, x_t)$
	□ $P(x_t\|x_{t-1})$	□ 1	

Solution:

The **observation update** expression has the following form:

$P(x_t|s_{1:t}, g_{1:t}) =$ **(iv)** **(v)** **(vi)** $P(x_t|s_{1:t-1}, g_{1:t-1})$.

Complete the expression by choosing the option that fills in each blank.

(iv)	□ $P(s_{1:t-1}\|s_t)P(g_{1:t-1}\|g_t)$	□ $\frac{1}{P(s_{1:t-1}\|s_t)P(g_{1:t-1}\|g_t)}$
	□ $P(s_t, g_t\|s_{1:t-1}, g_{1:t-1})$	□ $\frac{1}{P(s_{1:t-1}, g_{1:t-1}\|s_t, g_t)}$
	□ $\frac{1}{P(s_t\|s_{1:t-1})P(g_t\|g_{1:t-1})}$	□ $P(s_t\|s_{1:t-1})P(g_t\|g_{1:t-1})$
	□ $\frac{1}{P(s_t, g_t\|s_{1:t-1}, g_{1:t-1})}$	□ 1
	□ $P(s_{1:t-1}, g_{1:t-1}\|s_t, g_t)$	
(v)	□ $\sum_{x_t}$	□ $max_{x_{t-1}}$
	□ $\sum_{x_{t-1}}$	□ 1
	□ max_{x_t}	
(vi)	□ $P(x_{t-1}, s_{t-1})P(x_{t-1}, g_{t-1})$	□ $P(s_t\|x_t)P(g_t\|x_t)$
	□ $P(s_{t-1}\|x_{t-1})P(g_{t-1}\|x_{t-1})$	□ $P(x_t\|s_t)P(x_t\|g_t)$
	□ $P(x_{t-1}\|s_{t-1})P(x_{t-1}\|g_{t-1})$	□ $P(x_t, s_t, g_t)$
	□ $P(x_{t-1}, s_{t-1}, g_{t-1})$	□ 1
	□ $P(x_t, s_t)P(x_t, g_t)$	

Solution: