1 CSPs: Properties

a) In a general CSP with $n$ variables, each taking $d$ possible values, what is the maximum number of times a backtracking search algorithm might have to backtrack (i.e. the number of the times it generates an assignment, partial or complete, that violates the constraints) before finding a solution or concluding that none exists?

0 $O(1)$ $O(nd^2)$ $O(n^2d^3)$ $O(d^n)$

b) What is the maximum number of times a backtracking search algorithm might have to backtrack in a general CSP, if it is running arc consistency and applying the MRV and LCV heuristics?

0 $O(1)$ $O(nd^2)$ $O(n^2d^3)$ $O(d^n)$

c) What is the maximum number of times a backtracking search algorithm might have to backtrack in a tree-structured CSP, if it is running arc consistency and using an optimal variable ordering?

0 $O(1)$ $O(nd^2)$ $O(n^2d^3)$ $O(d^n)$
2 Constraint Graph

Consider the following constraint graph:

In two sentences or less, describe a strategy for efficiently solving a CSP with this constraint structure.

3 CSP: Air Traffic Control

We have five planes: A, B, C, D, and E and two runways: international and domestic. We would like to schedule a time slot and runway for each aircraft to either land or take off. We have four time slots: \{1, 2, 3, 4\} for each runway, during which we can schedule a landing or take off of a plane. We must find an assignment that meets the following constraints:

- Plane B has lost an engine and must land in time slot 1.
- Plane D can only arrive at the airport to land during or after time slot 3.
- Plane A is running low on fuel but can last until at most time slot 2.
- Plane D must land before plane C takes off, because some passengers must transfer from D to C.
- No two aircrafts can reserve the same time slot for the same runway

a) Complete the formulation of this problem as a CSP in terms of variables, domains, and constraints (both unary and binary). Constraints should be expressed implicitly using mathematical or logical notation rather than with words.

b) For the following subparts, we add the following two constraints:
Planes A, B, and C cater to international flights and can only use the international runway.

Planes D and E cater to domestic flights and can only use the domestic runway.

With the addition of the two constraints above, we completely reformulate the CSP. You are given the variables and domains of the new formulation. Complete the constraint graph for this problem given the original constraints and the two added ones. What are the domains of the variables after enforcing arc-consistency? Begin by enforcing unary constraints. (Cross out values that are no longer in the domain.)

| A | 1 2 3 4 |
| B | 1 2 3 4 |
| C | 1 2 3 4 |
| D | 1 2 3 4 |
| E | 1 2 3 4 |

Arc-consistency can be rather expensive to enforce, and we believe that we can obtain faster solutions using only **forward-checking** on our variable assignments. Using the Minimum Remaining Values heuristic, perform backtracking search on the graph, breaking ties by picking lower values and characters first. List the (variable, assignment) pairs in the order they occur (including the assignments that are reverted upon reaching a dead end). Enforce unary constraints before starting the search.

c) Suppose we have just one runway and \( n \) planes, where no two planes can use the runway at once. We are assured that the constraint graph will always be tree-structured and that a solution exists. What is the runtime complexity in terms of the number of planes, \( n \), of a CSP solver that runs arc-consistency and then assigns variables in a topological ordering?