Introduction to Information Retrieval

http://informationretrieval.org

IIR 21: Link Analysis

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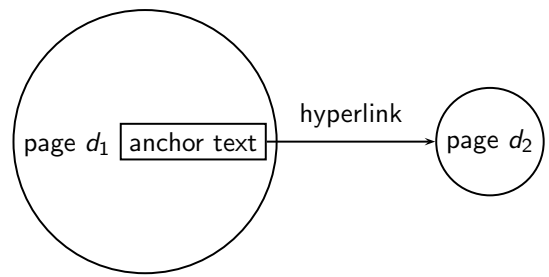
Overview

Outline

The web as a directed graph

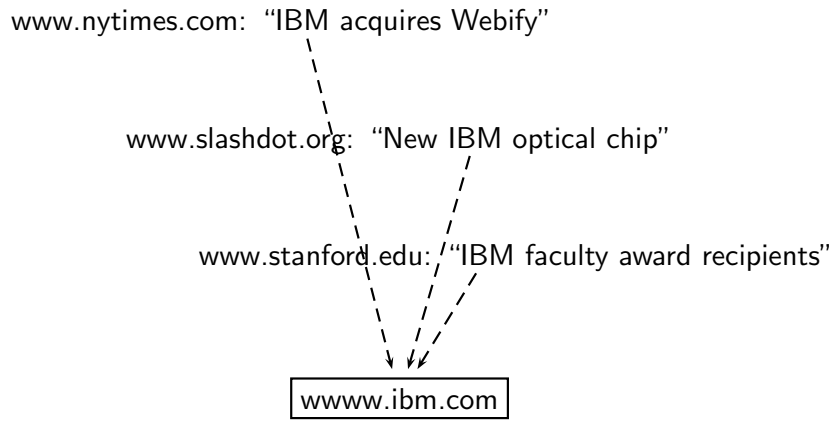


- Assumption 1: A hyperlink is a quality signal.
  - A hyperlink between pages denotes that the author perceived relevance.
- Assumption 2: The anchor text describes the target page d_2.
  - We use anchor text somewhat loosely here: the text surrounding the hyperlink. Example: "You can find cheap cars here."
- Examples for hyperlinks that violate these two assumptions?

[document text only] vs. [document text + anchor text]

- Searching on [document text + anchor text] is often more effective than searching on [document text only].
- Example: Query *IBM*
  - Matches IBM's copyright page
  - Matches many spam pages
  - Matches IBM wikipedia article
  - May not match IBM home page! (if IBM home page is mostly graphical)
- Searching on anchor text is better for the query *IBM*.
- Represent each page by all the anchor text pointing to it.
- In this representation, the page with the most occurrences of *IBM* is www.ibm.com.

Anchor text containing *IBM* pointing to www.ibm.com



Indexing anchor text

- Thus: Anchor text is often a better description of a page's content than the page itself.
- Anchor text can be weighted more highly than document text. (based on Assumptions 1&2)
- Indexing anchor text can have unexpected side effects – Google bombs.
- A Google bomb is a search with "bad" results due to maliciously manipulated anchor text.
- Google introduced a new weighting function in January 2007 that fixed many google bombs.
- Any "live" Google bombs?

Google bomb

- "who is a failure" on Google

Outline

Origins of PageRank: Citation analysis (1)

- Citation analysis: analysis of citations in the scientific literature
- Example citation: "Miller (2001) has shown that physical activity alters the metabolism of estrogens."
- "Miller (2001)" is a hyperlink linking two scientific articles.
- One application of these "hyperlinks" in the scientific literature:
  - Measure the similarity of two articles by the overlap of other articles citing them.
  - This is called cocitation similarity.
- Cocitation similarity on the web?

Cocitation similarity on Google: similar pages

Origins of PageRank: Citation analysis (2)

- Citation frequency can be used to measure the impact of an article.
  - Each article gets one vote.
  - Not a very accurate measure
- Better measure: weighted citation frequency / citation rank
  - An article's vote is weighted according to its citation impact.
  - Circular? No: can be formalized in a well-defined way.
  - This is basically PageRank.
  - PageRank was invented in the context of citation analysis by Pinsker and Narin in the 1960s.
  - Citation analysis is a big deal: The budget and salary of this lecturer are / will be determined by the impact of his publications!
- Recall: Citation in scientific literature = hyperlink on the web

Link-based ranking for web search

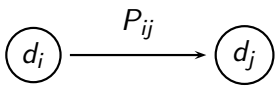
- Simple version of using links for ranking on the web
  - First retrieve all pages satisfying the query (say *venture capital*)
  - Order these by the number of in-links
- Simple link popularity (= number of in-links) is easy to spam. Why?

Basis for PageRank: Random walk

- Imagine a web surfer doing a random walk on the web
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably
- In the steady state, each page has a long-term visit rate.
- This long-term visit rate is the page's PageRank.
- PageRank = steady state probability = long-term visit rate
- Concept of long-term visit rate clear?

Markov chains

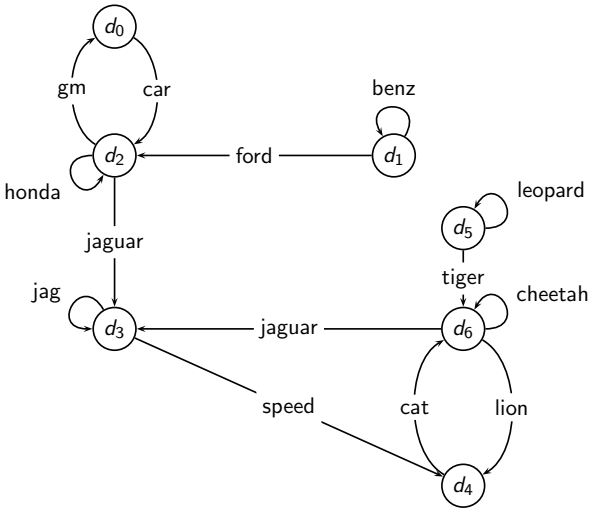
- A Markov chain consists of N states, plus an $N \times N$ transition probability matrix P.
- state = page
- At each step, we are on exactly one of the pages.
- For $1 \leq i, j \leq N$, the matrix entry P_{ij} tells us the probability of j being the next page, given we are currently on page i.



Markov chains

- Clearly, for all i, $\sum_{j=1}^{N} P_{ij} = 1$
- Markov chains are abstractions of random walks.

Example web graph



Link matrix for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	1	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
d_6	0	0	0	1	1	0	1

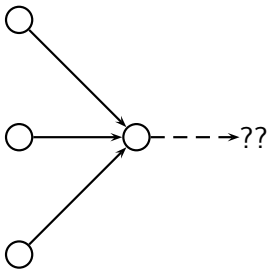
Transition probability matrix P for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Long-term visit rate

- Recall: PageRank = long-term visit rate
- Long-term visit rate of page d is the probability that a web surfer is at page d at a given point in time.
- Next: what properties must hold of the web graph for the long-term visit rate to be well defined?
- The web graph must correspond to an ergodic Markov chain.
- First a special case: The web graph must not contain dead ends.

Dead ends



- The web is full of dead ends.
- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

Teleporting

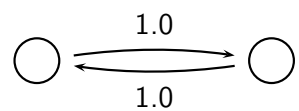
- At a dead end, jump to a random web page
- At any non-dead end, with probability 10%, jump to a random web page
- With remaining probability (90%), go out on a random hyperlink (e.g., randomly choose with probability (1-0.1)/4=0.225 one of the four hyperlinks of the page)
- 10% is a parameter.

Result of teleporting

- With teleporting, we cannot get stuck in a dead end.
- Concept of teleporting clear?
- Even without dead-ends, a graph may not have well-defined long-term visit rates.
- More generally, we require that the Markov chain be ergodic.

Ergodic Markov chains

- A Markov chain is ergodic iff it is irreducible and aperiodic.
- Irreducibility. Roughly: there is a path from any page to any other page.
- Aperiodicity. Roughly: The pages cannot be partitioned such that the random walker visits the partitions sequentially.



- A non-ergodic Markov chain:

Ergodic Markov chains

- Theorem: For any ergodic Markov chain, there is a unique long-term visit rate for each state.
- This is the steady-state probability distribution.
- Over a long time period, we visit each state in proportion to this rate.
- It doesn't matter where we start.

Formalization of "visit": Probability vector

- A probability (row) vector $\vec{x} = (x_1, \ldots, x_N)$ tells us where the random walk is at any point.
- Example:

(	0	0	0	...	1	...	0	0	0	)
	1	2	3	...	i	...	N-2	N-1	N	

- More generally: the random walk is on page i with probability x_i.
- Example:

(	0.05	0.01	0.0	...	0.2	...	0.01	0.05	0.03	)
	1	2	3	...	i	...	N-2	N-1	N	

- $\sum x_i = 1$

Change in probability vector

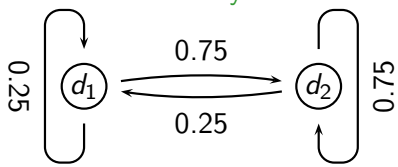
- If the probability vector is $\vec{x} = (x_1, \ldots, x_N)$ at this step, what is it at the next step?
- Recall that row i of the transition probability matrix P tells us where we go next from state i.
- Equivalently: column j of P tells us "where we came from" (and with which probability).
- So from $\vec{x}$, our next state is distributed as $\vec{x}P$.

Steady state in vector notation

- The steady state in vector notation is simply a vector $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_N)$ of probabilities.
- (We use $\vec{\pi}$ to distinguish it from the generic notation $\vec{x}$ for a probability vector.)
- π_i is the long-term visit rate (or PageRank) of page i.
- So we can think of PageRank as a very long vector – one entry per page.

Steady state example

- What is the steady state in this example?



- Solution: $\vec{\pi} = (\pi_1\ \pi_2) = (0.25\ 0.75)$

How do we compute the steady state vector?

- In other words: how do we compute PageRank?
- Recall: $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_N)$ is the PageRank vector, the vector of steady-state probabilities ...
- ... and if the distribution in this step is described by $\vec{x}$, then the distribution in the next step is distributed as $\vec{x}P$.
- But $\vec{\pi}$ is the steady state! So: $\vec{\pi} = \vec{\pi}P$
- Solving this matrix equation gives us $\vec{\pi}$.
- $\vec{\pi}$ is the principal left eigenvector for P ...
- ... that is, $\vec{\pi}$ is the left eigenvector with the largest eigenvalue.
- Transition probability matrices always have largest eigenvalue 1.

One way of computing the PageRank $\vec{\pi}$

- Recall: regardless of where we start (except for pathological cases), we eventually reach the steady state $\vec{\pi}$.
- Start with (almost) any distribution $\vec{x}$, e.g., uniform distribution
- After one step, we're at $\vec{x}P$.
- After two steps, we're at $\vec{x}P^2$.
- After k steps, we're at $\vec{x}P^k$.
- Algorithm: multiply $\vec{x}$ by increasing powers of P until the product looks stable.
- This is called the power method.

Power method: Example

- Two-node example: $\vec{x} = (0.5, 0.5)$, $P = \begin{pmatrix} 0.25 & 0.75 \\ 0.25 & 0.75 \end{pmatrix}$
- $\vec{x}P = (0.25, 0.75)$
- $\vec{x}P^2 = (0.25, 0.75)$
- Convergence in one iteration!

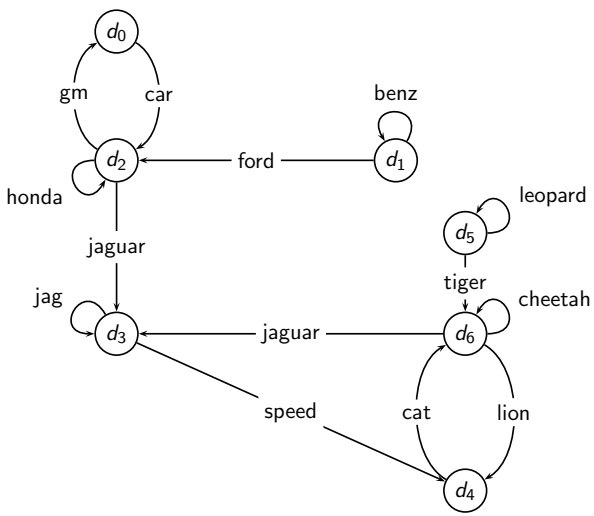
PageRank summary

- Preprocessing
  - Given graph of links, build matrix P
  - Apply teleportation
  - From modified matrix, compute $\vec{\pi}$
  - $\vec{\pi}_i$ is the PageRank of page i.
- Query processing
  - Retrieve pages satisfying the query
  - Rank them by their PageRank
  - Return reranked list to the user

PageRank issues

- Real surfers are not random surfers – Markov model is not a good model of surfing.
  - Issues: back button, short vs. long paths, bookmarks, directories – and search!
- Simple PageRank ranking (as described on previous slide) produces bad results for many pages.
  - Consider the query *video service*
  - The Yahoo home page (i) has a very high PageRank and (ii) contains both words.
  - If we rank all Boolean hits according to PageRank, then the Yahoo home page would be top-ranked.
  - Clearly not desirable
- In practice: rank according to weighted combination of many factors, including raw text match, anchor text match, PageRank and many other factors

Web graph example



Transition (probability) matrix

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Transition matrix with teleporting

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.02	0.02	0.88	0.02	0.02	0.02	0.02
d_1	0.02	0.45	0.45	0.02	0.02	0.02	0.02
d_2	0.31	0.02	0.31	0.31	0.02	0.02	0.02
d_3	0.02	0.02	0.02	0.45	0.45	0.02	0.02
d_4	0.02	0.02	0.02	0.02	0.02	0.02	0.88
d_5	0.02	0.02	0.02	0.02	0.02	0.45	0.45
d_6	0.02	0.02	0.02	0.31	0.31	0.02	0.31

Power method vectors $\vec{x}P^k$

	$\vec{x}$	$\vec{x}P^1$	$\vec{x}P^2$	$\vec{x}P^3$	$\vec{x}P^4$	$\vec{x}P^5$	$\vec{x}P^6$	$\vec{x}P^7$	$\vec{x}P^8$	$\vec{x}P^9$	$\vec{x}P^{10}$	$\vec{x}P^{11}$	$\vec{x}P^{12}$	$\vec{x}P^{13}$
d_0	0.14	0.06	0.09	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
d_1	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_2	0.14	0.25	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11
d_3	0.14	0.16	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25
d_4	0.14	0.12	0.16	0.19	0.19	0.20	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
d_5	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_6	0.14	0.25	0.23	0.25	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.31	0.31

How important is PageRank?

- Frequent claim: PageRank is the most important component of web ranking.
- The reality:
  - There are several components that are at least as important: e.g., anchor text, phrases, proximity, tiered indexes ...
  - Rumor has it that PageRank in its original form (as presented here) has a negligible impact on ranking!
  - However, variants of a page's PageRank are still an essential part of ranking.
  - Adressing link spam is difficult and crucial.

Outline

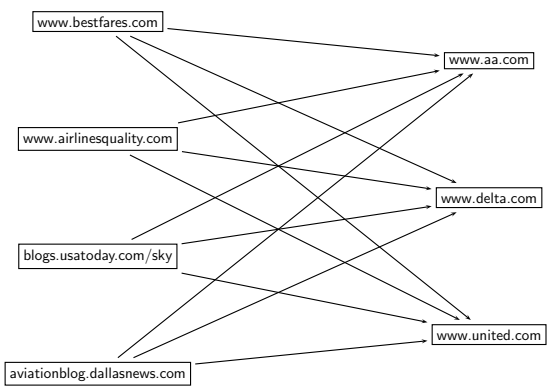
HITS – Hyperlink-Induced Topic Search

- Premise: there are two different types of relevance on the web.
- Relevance type 1: Hubs. A hub page is a good list of links to pages answering the information need.
  - Bob's list of recommended hotels in London
- Relevance type 2: Authorities. An authority page is a direct answer to the information need. Authority pages occur repeatedly on hub pages.
  - Home page of Four Seasons Hotel London
- Most approaches to search (including PageRank ranking) don't make the distinction between these two very different types of relevance.

Hubs and authorities

- A good hub page for a topic points to many authority pages for that topic.
- A good authority page for a topic is pointed to by many hub pages for that topic.
- Circular definition – we will turn this into an iterative computation.

Example for hubs and authorities

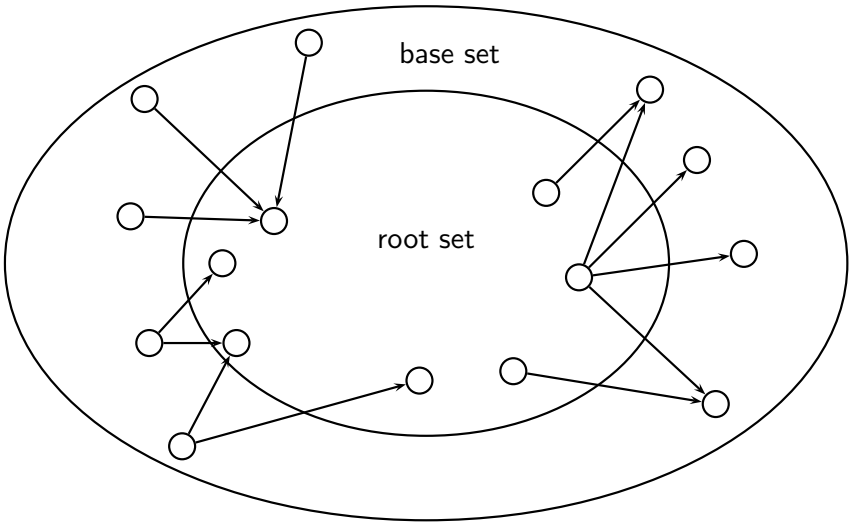


Definition clear?

Root set and base set (1)

- Do a regular web search first
- Call the search result the root set
- Find all pages that are linked to or link to pages in the root set
- Call this larger set the base set
- Finally, compute hubs and authorities for this (small) web graph

Root set and base set (2)



Root set and base set (3)

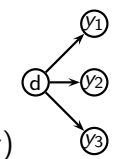
- Root set typically has 200–1000 nodes.
- Base set may have up to 5000 nodes.
- Computation of base set:
  - Follow out-links by parsing the pages in the root set
  - Find d's in-links by searching for all pages containing a link to d
  - This assumes that our inverted index supports search for links (in addition to terms).

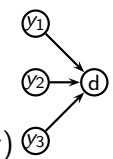
Hub and authority scores

- Compute for each page d in the base set a hub score $h(d)$ and an authority score $a(d)$
- Initialization: for all d: $h(d) = 1$, $a(d) = 1$
- Iteratively update all $h(d), a(d)$
- After convergence:
  - Output pages with highest h scores as top hubs
  - Output pages with highest a scores as top authorities
  - So we output two ranked lists

Iterative update

- For all d: $h(d) = \sum_{d \mapsto y} a(y)$



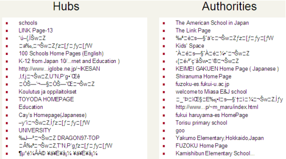


- For all d: $a(d) = \sum_{y \mapsto d} h(y)$
- Iterate these two steps until convergence

Details

- Scaling
  - To prevent the $a()$ and $h()$ values from getting too big, can scale down after each iteration
  - Scaling factor doesn't really matter.
  - We care about the relative (as opposed to absolute) values of the scores.
- In most cases, the algorithm converges after a few iterations.

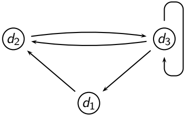
Japan elementary schools



- The query was "Japan elementary schools".
- HITS pulled together good pages regardless of page content.
- An English query was able to retrieve Japanese-language pages!
- Once the base set is assembled, we only do link analysis, no text matching.
- Danger: topic drift – the pages found by following links may not be related to the original query.

Proof of convergence

- We define an $N \times N$ adjacency matrix A.
- For $1 \leq i, j \leq N$, the matrix entry A_{ij} tells us whether there is a link from page i to page j ($A_{ij} = 1$) or not ($A_{ij} = 0$).
- Example:



	d_1	d_2	d_3
d_1	0	1	0
d_2	1	1	1
d_3	1	0	0

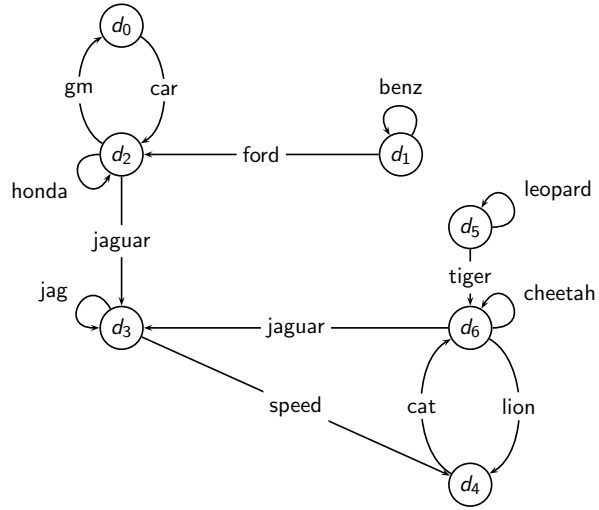
Write update rules as matrix operations

- Define the hub vector $\vec{h} = (h_1, \ldots, h_N)$ as the vector of hub scores. h_i is the hub score of page d_i.
- Similarly for $\vec{a}$, the vector of authority scores
- Now we can write $h(d) = \sum_{d \mapsto y} a(y)$ as a matrix operation: $\vec{h} = A\vec{a}$ ...
- ... and we can write $a(d) = \sum_{y \mapsto d} h(y)$ as $\vec{a} = A^T\vec{h}$
- HITS algorithm in matrix notation:
  - Compute $\vec{h} = A\vec{a}$
  - Compute $\vec{a} = A^T\vec{h}$
  - Iterate until convergence

HITS as eigenvector problem

- HITS algorithm in matrix notation. Iterate:
  - Compute $\vec{h} = A\vec{a}$
  - Compute $\vec{a} = A^T\vec{h}$
- By substitution we get: $\vec{h} = AA^T\vec{h}$ and $\vec{a} = A^TA\vec{a}$
- Thus, $\vec{h}$ is an eigenvector of AA^T and $\vec{a}$ is an eigenvector of A^TA.
- So the HITS algorithm is actually a special case of the power method and hub and authority scores are eigenvector values.
- HITS and PageRank both formalize link analysis as eigenvector problems.

Example web graph



Raw matrix H for HITS

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	2	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
d_6	0	0	0	2	1	0	1

Hub vectors $h_0, \vec{h}_i = \frac{1}{d_i} H \cdot \vec{a}_i, i \geq 1$

	$\vec{h}_0$	$\vec{h}_1$	$\vec{h}_2$	$\vec{h}_3$	$\vec{h}_4$	$\vec{h}_5$
d_0	0.14	0.06	0.04	0.04	0.03	0.03
d_1	0.14	0.08	0.05	0.04	0.04	0.04
d_2	0.14	0.28	0.32	0.33	0.33	0.33
d_3	0.14	0.14	0.17	0.18	0.18	0.18
d_4	0.14	0.06	0.04	0.04	0.04	0.04
d_5	0.14	0.08	0.05	0.04	0.04	0.04
d_6	0.14	0.30	0.33	0.34	0.35	0.35

Authority vectors $\vec{a}_i = \frac{1}{c_i} H^T \cdot \vec{h}_{i-1}, i \geq 1$

	$\vec{a}_1$	$\vec{a}_2$	$\vec{a}_3$	$\vec{a}_4$	$\vec{a}_5$	$\vec{a}_6$	$\vec{a}_7$
d_0	0.06	0.09	0.10	0.10	0.10	0.10	0.10
d_1	0.06	0.03	0.01	0.01	0.01	0.01	0.01
d_2	0.19	0.14	0.13	0.12	0.12	0.12	0.12
d_3	0.31	0.43	0.46	0.46	0.46	0.47	0.47
d_4	0.13	0.14	0.16	0.16	0.16	0.16	0.16
d_5	0.06	0.03	0.02	0.01	0.01	0.01	0.01
d_6	0.19	0.14	0.13	0.13	0.13	0.13	0.13

Top-ranked pages

- Pages with highest in-degree: d_2, d_3, d_6
- Pages with highest out-degree: d_2, d_6
- Pages with highest PageRank: d_6
- Pages with highest hub score: d_6 (close: d_2)
- Pages with highest authority score: d_3

PageRank vs. HITS: Discussion

- PageRank can be precomputed, HITS has to be computed at query time.
  - HITS is too expensive in most application scenarios.
- The PageRank and HITS make two different design choices concerning (i) the eigenproblem formalization (ii) the set of pages to apply the formalization to.
- These two are orthogonal.
  - We could also apply HITS to the entire web and PageRank to a small base set.
- On the web, a good hub almost always is also a good authority.
- Why?
- The actual difference between PageRank ranking and HITS ranking is therefore not as large as one might expect.

Resources

- Chapter 21 of IIR
- Resources at `http://ifnlp.org/ir`
- American Mathematical Society article on PageRank (popular science style)
- Jon Kleinberg's home page (main person behind HITS)
- Google's official description of PageRank: *PageRank reflects our view of the importance of web pages by considering more than 500 million variables and 2 billion terms. Pages that we believe are important pages receive a higher PageRank and are more likely to appear at the top of the search results.*